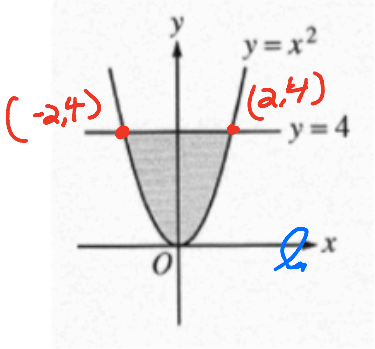
Period 1, May 6, 2024

Applications of Integrals Free Response Area/Volume
Graphing Calculator is permitted

$x^2 = y$

$x^2 = 4$

$x = 2, -2$



1. The shaded region, R, is bounded by the graph of $y = x^2$ and the line $y = 4$, as shown in the figure above.

(a) Find the area of R.

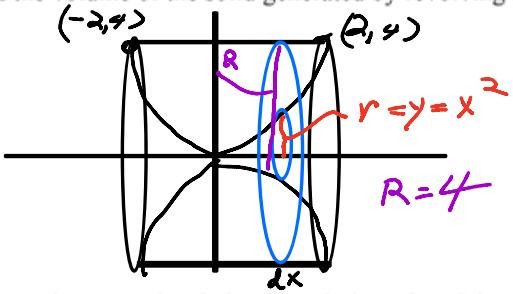
$$\int_{-2}^{2} (4 - x^2)\,dx = 4x - \frac{1}{3}x^3 + C$$

$$4(2) - \frac{1}{3}(2)^3 - \left[4(-2) - \frac{1}{3}(-2)^3\right]$$

$$8 - \frac{8}{3} + 8 - \frac{8}{3} = 16 - \frac{16}{3} = \frac{48}{3} - \frac{16}{3} = \frac{32}{3}$$

$10\frac{2}{3}$

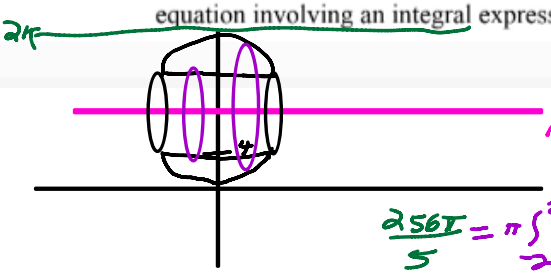
(b) Find the volume of the solid generated by revolving R about the x-axis.



$$\pi \int_{-2}^{2} (4^2 - (x^2)^2)\,dx$$

$$\pi\left(16x - \frac{1}{5}x^5\right)\Big|_{-2}^{2} = \frac{256\pi}{5}$$

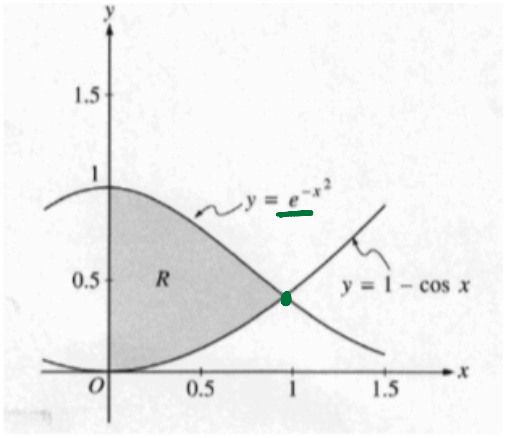
(c) There exists a number k, $k > 4$, such that when R is revolved about the line $y = k$, the resulting solid has the same volume as the solid in part (b). Write, but do not solve, an equation involving an integral expression that can be used to find the value of k.



$r = k - 4$

$R = k - x^2$

$$\frac{256\pi}{5} = \pi \int_{-2}^{2} \left((k - x^2)^2 - (k-4)^2\right)dx$$



$1-\cos x = e^{-x^2}$

$y = 1-\cos x$

$y = e^{-x^2}$

intersect

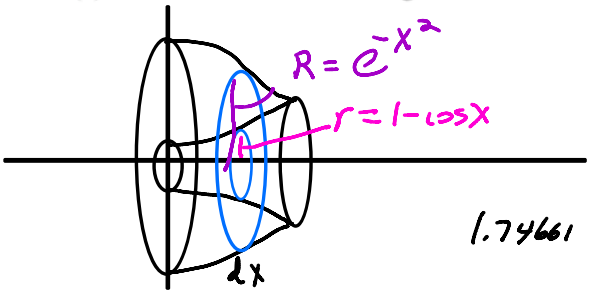
.941944, .412

2. Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$ and the y-axis, as shown in the figure above.

(a) Find the area of the region R.

$$\int_0^{.941944} (e^{-x^2} - (1-\cos x))\,dx = 0.59$$

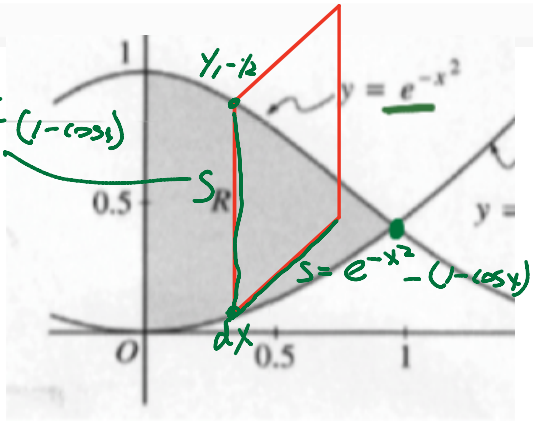
(b) Find the volume of the solid generated when the region R is revolved about the x-axis.



$$\pi \int_0^{.941944} \left((e^{-x^2})^2 - (1-\cos x)^2\right) dx$$

$$1.74661 = \pi \int_0^{.941944} \left[e^{-2x^2} - 1 + 2\cos x - \cos^2 x\right] dx$$

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.



$e^{-x^2} - (1-\cos x)$

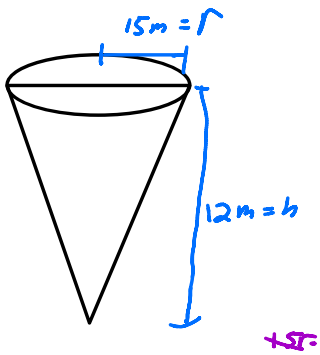
$$\int_0^{.941944} \left(e^{-x^2} - (1-\cos x)\right)^2 dx$$

0.46104

$$\int_0^{.941944} \left(e^{\left(-x^2\right)} - (1-\cos x)\right)^2 dx$$

$$= 0.461063510705$$

10.



$\frac{dV}{dT} = 2 \frac{m^3}{min}$

$\frac{dh}{dT} = ?$ when $h = 8$ meters

$\frac{12}{15} = \frac{h}{r}$

$12r = 15h$

$r = \frac{5}{4}h$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \left(\frac{5}{4}h\right)^2 \cdot h$

$V = \frac{1}{3}\pi \cdot \frac{25}{16}h^2 \cdot h$

$V = \frac{25\pi}{48}h^3 \Rightarrow \frac{dV}{dT} = \frac{25\pi}{48} \cdot 3h^2 \frac{dh}{dT}$

$2 \frac{m^3}{min} = \frac{25\pi}{48}(3(8m)^2)\frac{dh}{dT}$

$2 \frac{m^3}{min} = \frac{25\pi}{3 \cdot 16} \cdot 3 \cdot 8 \cdot 8\, m^2 \frac{dh}{dT}$

$\frac{2 m^3/min}{100\pi m^2} = \frac{100\pi m^2}{100\pi m^2}\frac{dh}{dT}$

$\frac{2 m^3}{min} \cdot \frac{1}{100\pi m^2} = \frac{dh}{dT} \Rightarrow \frac{1}{50\pi} \frac{m}{min}$

$V = \frac{1}{3}\pi r^2 h$

$\frac{dV}{dT} = \frac{1}{3}\pi \cdot 2r \cdot \frac{dr}{dT} \cdot h + \frac{1}{3}\pi r^2 \cdot \frac{dh}{dT}$

$2 = \frac{1}{3}\pi \cdot 2 \cdot 10 \cdot \frac{5}{4}\frac{dh}{dT} \cdot 8 + \frac{1}{3} \cdot \pi \cdot 100 \frac{dh}{dT}$

$12r = 15h$

$12\frac{dr}{dT} = 15\frac{dh}{dT}$

$\frac{dr}{dT} = \frac{15}{12}\frac{dh}{dT} = \frac{5}{4}\frac{dh}{dT}$

$h = 8 \quad r =$

$12r = 15 \cdot 8 = 120$

$r = 10$

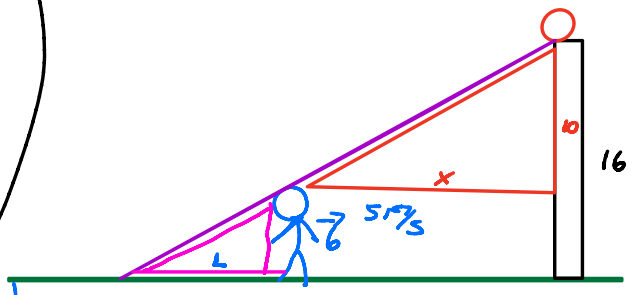
$2 = \frac{800\pi}{12}\frac{dh}{dT} + \frac{400\pi}{12} \cdot \frac{dh}{dT}$

$2 = \frac{1200\pi}{12}\frac{dh}{dT}$

$2 = 100\pi \frac{dh}{dT}$

$\frac{2}{100\pi} = \frac{1}{50\pi} = \frac{dh}{dT}$

9.



Shadow

$\frac{dx}{dT} = -5 \frac{FT}{Sec}$

$\frac{L}{6} = \frac{x}{10}$

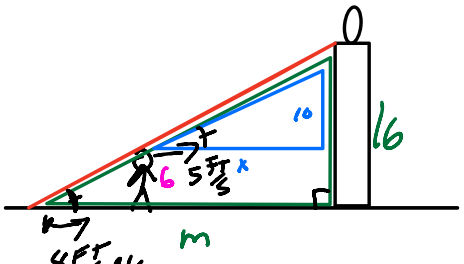
$10L = 6x$

$10\frac{dL}{dT} = 6\frac{dx}{dT}$

$10\frac{dL}{dT} = 6 \cdot -5 \frac{FT}{S}$

$\frac{10\frac{dL}{dT}}{10} = \frac{-30 \frac{FT}{S}}{10}$

$\frac{dL}{dT} = -3 \frac{FT}{Sec}$



$\frac{10}{x} = \frac{16}{m}$

$16x = 10m$

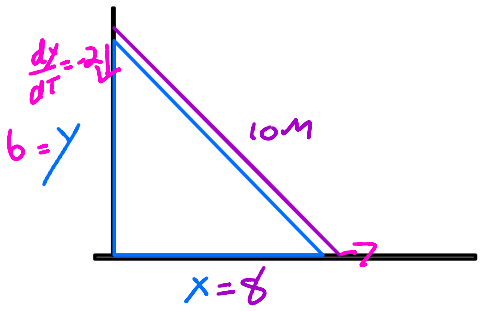
$16\frac{dx}{dT} = 10\frac{dm}{dT}$

$\frac{16 \cdot -5}{10} = \frac{10\frac{dm}{dT}}{10}$

$\frac{dx}{dT} = -5 \frac{FT}{Sec}$

$-8 \frac{FT}{S} = \frac{dm}{dT}$

$\frac{dm}{dT} - \frac{dx}{dT} = \frac{ds_h}{dT}$



$x^2 + 6^2 = 100 \Rightarrow x^2 + 36 = 100$

$x^2 = 64$

$x = \pm 8$

$x^2 + y^2 = 10^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2x\frac{dx}{dt} + 2(6)(-2\ m/s) = 0$

$2 \cdot 8 \cdot \frac{dx}{dt} + -24 = 0$

$16\frac{dx}{dt} = 24$

$\frac{dx}{dt} = \frac{24}{16} = +\frac{3}{2}\ m/s$